(1950). Apparently, it is not obvious that the simple scalar deformation potential model of Shockley and Bardeen is adequate. One should, in principle, formulate V in terms of a deformation-potential tensor. However, the effect of anisotropy is not large: ~1.5% (see e.g., Ref. 14, p. 115). In any case, the value of E_1 in our case is determined by the zero magnetic field experimental results. [See text following Eq. (4.18)].

[16]R. Kubo *et al.* (Ref. 6) have given a treatment of collision broadening and have compared this with the cutoff due to inelasticity. From these calculations, it may be verified that inelasticity of acoustic-phonon scattering would be present at sufficiently low temperatures (~15 °K) while collision broadening is ineffective, except at very high temperatures (>100 °K) and very high magnetic fields (>300 kG). In the 30-77 °K, 0–200-kG region of interest to us, the cutoff factor found naturally in the theory should be adequate for strong as well as weak magnetic fields. The present theory can be extended to inelastic phonon scattering as well.

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Quantum Oscillations of Microwave Helicon Dispersion in n-Type InSb and InAs†

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The effect of orbital quantization on microwave helicon dispersion in n-type InSb and InAs is investigated theoretically and experimentally. In the local limit, the leading term describing helicon dispersion is, unlike helicon damping, unaffected by orbital quantization. Quantum effects enter through the *scattering-dependent* terms involved in the dispersive part of the helicon propagation constant. The main contribution is shown to be associated with the Shubnikov–de Haas-like oscillations of the scattering correction to the dissipationless Hall conductivity. Experimental measurements of the transmitted helicon phase observed at quantizing magnetic fields in highly doped n-type InSb and InAs at 35 GHz and liquid-helium temperature are compared with the theory. The magnetic field dependence of the observed oscillations in helicon phase agrees reasonably well with the theoretical analysis. While little can be said analytically about the amplitude of these oscillations (of the order of a few percent), our data does provide an empirical measure of the limits within which the usual classical analysis of helicon dispersion is valid. Finally, the effect of quantum oscillations appears to be considerably stronger in the helicon dispersion than in the related dc problem of the Hall coefficient.

INTRODUCTION

It is well known that in the quantum limit helicon-wave damping in semiconductors and semimetals displays strong Shubnikov–de Haas-like oscillations. [1,2] At the same time it is usually assumed that helicon *dispersion* is free of the effects of orbital quantization. Of course, this is an approximation, since both damping and dispersion originate in the same conductivity tensor. The behavior of helicon damping and dispersion is similar, respectively, to the behavior of transverse dc magnetoresistance and Hall effect. While in magnetoresistance the Shubnikov–de Haas oscillations are overwhelming, the Hall coefficient is relatively independent of these contributions. Nevertheless, weak quantum oscillations in the Hall effect in semiconductors have been known qualitatively for some 15 years [3] and have more recently been a subject of quantitative experimental as well as theoretical study. [4,5]

In this paper we investigate the oscillatory magnetic field dependence of the local helicon-wave dispersion in small-gap semiconductors in the quantum limit. In order to determine the dominant oscillatory contributions to the helicon dispersion, we analyze the general expression for transmitted helicon phase in the light of existing theoretical formulations of the appropriate quantum conductivity tensor. Specifically, it is shown that, in the parameter range of interest, oscillatory contributions to the helicon dispersion originate primarily in the *frequency-independent* elements of the local conductivity tensor and are dominated by the small collision-dependent term in the dissipationless Hall conductivity. The main features of the effect are discussed, by way of illustration, for the region of high quantum numbers, for which a rigorous theory exists. The discussion is then extended to low quantum numbers corresponding to our experimental conditions. The extension to low quantum numbers is accomplished in part by semiquantitative plausibility arguments, because a rigorous analytic approach is impractical in this range.

We then compare our conclusions with a detailed analysis of experimental data obtained on doped n-type InSb and InAs, which reveal oscillations in the transmitted helicon phase of the order of a few percent. The magnetic field dependence of the oscillations agrees reasonably well with the analysis, and their amplitude provides an empirical measure of the extent to which the conventional classical expression can be used in describing helicon dispersion in quantizing fields.

We finally contrast the dependence of helicon phase on quantum contributions with the related dc problem of the Hall coefficient and find that, within the framework of our model, the neglect of oscillatory terms in the helicon problem is generally more serious.

THEORETICAL DETAILS

In order to estimate the oscillatory contributions to the transmitted helicon phase, we inspect the analytic expressions describing wave propagation in the helicon limit, i.e., in the parameter range defined by the inequality $\omega_p^2/\omega \gg \omega_c \gg \omega, \tau^{-1}$. Here $\omega_p \equiv (ne^2/m^*\epsilon_l)^{1/2}$ is the plasma frequency, ϵ_l is the static permittivity of the lattice, $\omega_c \equiv eB/m^*$ is the cyclotron frequency, τ^{-1} the phenomenological collision frequency, and n and m^* are the electron concentration and effective mass, respectively. MKS units will be used in this paper. Under these conditions, waves which are circularly polarized in the cyclotron-resonance-active (CRA) sense (i.e., helicon waves) are described by the complex propagation vector $k \equiv \alpha + i\beta$, with $\alpha \gg \beta$ and $\alpha \gg k_0$, where k_0 is the propagation constant in free space. We shall further restrict ourselves to the local limit, appropriate for semiconductor phenomena in high magnetic fields, and given by the inequality $kv_F/\omega_c \ll kv_F\tau \ll 1$, where v_F is the Fermi velocity.

For slab samples thicker than the skin depth, which correspond to our typical experimental conditions, the transmitted signal for plane waves is of the form

$$T = A_h e^{i\varphi_h} \simeq t_1 t_2 e^{ikz} \simeq (4k_0/\alpha) e^{-\beta z} e^{i(\alpha z - \beta/\alpha)}, \qquad (1)$$

where the explicit time dependence has been suppressed. Here A_h and φ_h are the transmitted helicon amplitude and phase, z is the sample thickness, and t_1 and t_2 are the transmission coefficients for the two interfaces, given by[6]

$$t_1 t_2 = \frac{4k_0 k}{(k_0+k)^2} \simeq \frac{4k_0}{k} = \frac{4k_0(\alpha - i\beta)}{\alpha^2+\beta^2}$$
$$= \frac{4k_0}{(\alpha^2+\beta^2)^{1/2}} e^{-i\arctan(\beta/\alpha)} \simeq \frac{4k_0}{\alpha} e^{-i\beta/\alpha}. \qquad (2)$$

Dispersion and damping coefficients are given in terms of the complex permittivity $\epsilon = \epsilon' + i\epsilon''$ (which in the helicon limit satisfies the small loss tangent condition $\epsilon'' \ll \epsilon'$) by

$$\alpha = \omega(\mu_0/2)^{1/2}(|\epsilon| + \epsilon')^{1/2} \simeq \omega\{\mu_0\epsilon'[1 + \tfrac{1}{4}(\epsilon''/\epsilon')^2]\}^{1/2}, \qquad (3a)$$

$$\beta = \omega(\mu_0/2)^{1/2}(|\epsilon| - \epsilon')^{1/2} \simeq \tfrac{1}{2}\omega(\mu_0/\epsilon')^{1/2}\epsilon'' \simeq \tfrac{1}{2}\alpha\epsilon''/\epsilon', \qquad (3b)$$

where the lowest-order terms in ϵ'', of direct significance in this discussion, have been retained.

For propagation of circularly polarized waves along an external magnetic field, the effective permittivities are conveniently expressed in terms of the components of the complex conductivity tensor $\sigma_{ij} \equiv \sigma'_{ij} + i\sigma''_{ij}$ as

$$\epsilon'_\pm = \epsilon_l + (\sigma''_{xx} \pm \sigma'_{xy})/\omega, \qquad (4a)$$

$$\epsilon''_\pm = (\sigma'_{xx} \mp \sigma''_{xy})/\omega, \qquad (4b)$$

where z is the direction of propagation and of the external magnetic field B, and the + and − subscripts refer to the two circularly polarized normal modes.

We now examine the effect of orbital quantization on ϵ' and ϵ'', and thus ultimately on the transmitted helicon phase φ_h, by studying σ_{ij}. The parameter range of interest here is $kT \ll \hbar\omega_c$, ζ_F, where ζ_F is the Fermi energy, as well as the condition $\omega_c \gg \tau^{-1}$, ω already implicit in the helicon limit.

The literature concerned with static magnetoconductivity of semiconductors and semimetals in the quantum regime is vast.[7] By comparison, little attention has been given to the problem of frequency-dependent magnetoconductivity in quantum plasmas *including effects of collisions* (which is of paramount importance in Shubnikov–de Haas oscillations).[8-11] While the results obtained by some of the authors are physically quite unrevealing and not readily tractable, we find the form of $\sigma_{ij}(\omega, \tau)$ obtained by Wolman and Ron[8] for degenerate quantum plasmas ideally suited for the present discussion.

We first examine the dissipative term ϵ'', which contributes a small correction to α in Eq. (3a). This quantity, directly related to β, is strongly oscillatory in the quantum regime, as can be seen from the behavior of helicon damping, cf., Fig. 1.

In the helicon limit it can be readily shown that $\sigma''_{xy}(\omega) \simeq 2(\omega/\omega_c)\sigma'_{xx}(\omega)$ and that, to first order in ω/ω_c, $\sigma'_{xx}(\omega) \simeq \sigma_{xx}(0)$. Thus, for $\omega \ll \omega_c$,

$$\epsilon''_{\pm} \approx \sigma_{xx}(0)/\omega, \tag{5}$$

i.e., the leading term in ϵ'' is determined by transverse dc magnetoconductivity. This is clearly born out by previous experiments, which indicate excellent correlation between helicon damping and the dc magnetoconductivity.[1,2]

In addition, there will be quantum contributions to the dissipationless quantity ϵ'. This quantity is, to first order in the expansion parameter $[(\omega_c \pm \omega)\tau]^{-1}$, independent of orbital quantization effects,[8] and higher-order terms must be examined for the oscillatory effects. We can write

$$\epsilon'_{\pm} \equiv \epsilon_l + \frac{\sigma''_{xx} \pm \sigma'_{xy}}{\omega} = \epsilon_l + \frac{\omega_p^2 \epsilon_l}{\omega} \frac{(\omega \pm \omega_c)}{(\omega^2 - \omega_c^2)} \times [1 + O((\omega_c \pm \omega)^{-2}\tau^{-2})], \tag{6}$$

which in the helicon limit becomes

$$\epsilon'_{\pm} \approx \mp \frac{ne}{\omega B}[1 + O(\omega_c \tau)^{-2}] = \mp \frac{\sigma_{xy}(0)}{\omega}. \tag{7}$$

Note that $\epsilon'_{-} > 0$. This corresponds to the propagating CRA or helicon polarization. Our attention will henceforth be restricted to this mode, and the subscripts on ϵ will be dropped.

Thus, the problem of determining the leading *oscillatory* term in ϵ' [i.e., the second-order correction in Eq. (7)] is also reduced to the dc case, and use can be made of existing theoretical results for the static Hall conductivity.[5] The error implicit in Eq. (7) amounts essentially to a negligible *monotonic* shift in B, of the order of ω/ω_c.

Equations (5) and (7) could have been written immediately on intuitive grounds for the leading terms of ϵ' and of ϵ'' if the inequality $\omega_c \gg \omega, \tau^{-1}$ is satisfied (as was done in Ref. 1). However, since in ϵ' the quantum-dependent contributions of interest to us appear only in the second order of the expansion parameter,[8] it was necessary to demonstrate that the effect of σ''_{xx} in Eq. (6), which contributes to ϵ' a frequency-dependent term of the order of ω/ω_c relative to the contribution of σ'_{xy}, is nonoscillatory. Even this can be done on semi-intuitive grounds by

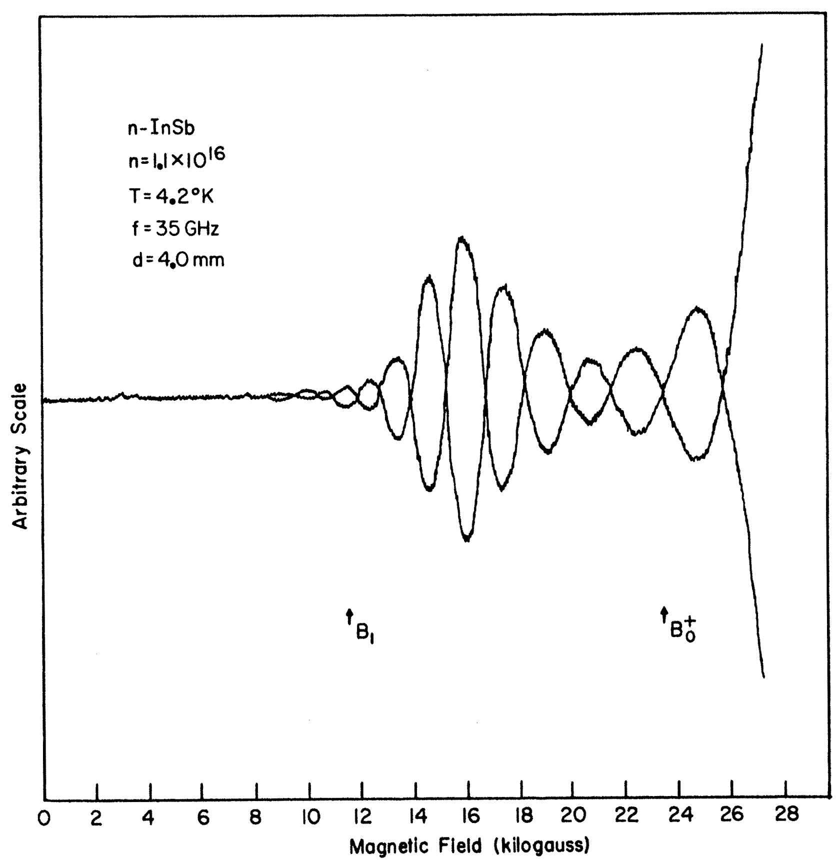

FIG. 1. Experimental data showing Rayleigh interference patterns of transmission through doped n-type InSb obtained at 35 GHz with two phase settings 180° out of phase. Theoretical values of B corresponding to singularities in σ_{xx}, calculated via Eq. (14), are indicated.

arguing that since the mechanism of the Shubnikov-de Haas oscillations is intimately associated with the scattering process, those elements of the conductivity tensor which in the *classical* form (e.g., the Drude-model expression for σ_{xx}'') are τ-independent, will be essentially unaffected by orbital quantization. The elegant result of Wolman and Ron essentially substantiates this convenient rule of thumb.

The oscillatory nature of magnetoconductivity arises primarily because the singularities in the density of states occurring at the Landau levels lead to oscillations in the scattering rates as the magnetic field is varied, i.e., as the successive Landau levels pass the Fermi energy. Following Adams and Holstein[12] (AH), we express the density of states $N(\zeta_F)$ in terms of oscillatory and non-oscillatory parts:

$$N(\zeta_F) \equiv N_0(\zeta_F)\left[1 + f_{\rm osc}(\zeta_F)\right], \tag{8a}$$

where

$$N_0 = 4\pi m(2m\zeta_F)^{1/2}/(2\pi\hbar)^3, \tag{8b}$$

$$f_{\rm osc}(\zeta_F) = (\hbar\omega_c/\zeta_F)^{1/2}\left[\tfrac{1}{2}\delta^{-1/2} - (\delta + \tfrac{1}{2})^{1/2}\right], \tag{8c}$$

$$\delta = (\zeta_F/\hbar\omega_c) - (I + \tfrac{1}{2}). \tag{8d}$$

Here I is the integer associated with the Landau level just below ζ_F, thus $0 < \delta < 1$. The singularity in Eq. (8c) at $\delta = 0$ can be removed by imposing a lower limit on δ through broadening effects.[12] Thus for collisional broadening $\delta_{min} \simeq \hbar\tau^{-1}/\hbar\omega_c = (\omega_c\tau)^{-1}$. In terms of the above, AH obtain for the diagonal conductivity

$$\sigma_{xx} = (\sigma_{xx})_{cl} + (\sigma_{xx})_{\rm osc}, \tag{9}$$

where

$$(\sigma_{xx})_{cl} = (ne/B)(1/\omega_c\tau_0),$$

$$(\sigma_{xx})_{\rm osc} = (\sigma_{xx})_{cl}\left[\tfrac{5}{2}f_{\rm osc} + \tfrac{3}{2}(f_{\rm osc})^2\right],$$

and τ_0 is the relaxation time defined within the Born approximation, which also satisfies the condition $\omega_c\tau_0 \gg 1$ in our range of interest.

Proceeding similarly,[13] Guseva and Zyryanov (GZ) have recently investigated the effect of orbital quantization on the Hall conductivity, obtaining

$$\sigma_{xy} = \sigma_{xy}^{(0)}(1 - \Delta_{xy}) = \sigma_{xy}^{(0)}\left[1 - (\Delta_{xy})_{cl} - (\Delta_{xy})_{\rm osc}\right], \tag{10}$$

with

$$\sigma_{xy}^{(0)} = ne/B, \qquad (\Delta_{xy})_{cl} = (\omega_c\tau_0)^{-2},$$

$$(\Delta_{xy})_{\rm osc} = (\Delta_{xy})_{cl}\left[\tfrac{7}{2}f_{\rm osc} + 4(f_{\rm osc})^2 + \tfrac{3}{2}(f_{\rm osc})^3\right],$$

where $\sigma_{xy}^{(0)} \equiv ne/B$ is the ideal (collisionless) Hall conductivity. We thus observe that the oscillatory correction to σ_{xy} is of the order of $[\sigma_{xx}/\sigma_{xy}^{(0)}]^2 \approx (\omega_c\tau_0)^{-2}$ relative to the leading term.

We finally express the transmitted helicon phase in terms of the above results, retaining the lowest-order oscillatory contributions of ϵ' and ϵ'' to φ_h:

$$\varphi_h = \alpha z - \frac{\beta}{\alpha} = z(\omega\mu_0\sigma_{xy})^{1/2}\left[1 + \frac{1}{4}\left(\frac{\sigma_{xx}}{\sigma_{xy}}\right)^2\right]^{1/2} - \frac{\beta}{\alpha}$$

$$\simeq z\left[\omega\mu_0\sigma_{xy}^{(0)}\right]^{1/2}\left[1 - \Delta_{xy} + \frac{1}{4}\left(\frac{\sigma_{xx}}{\sigma_{xy}^{(0)}}\right)^2\right]^{1/2} - \frac{\beta}{\alpha_0}$$

$$\simeq z\alpha_0\left[1 - \tfrac{1}{2}\Delta_{xy} + \tfrac{1}{8}\Delta_{xx}(1 - 2/\beta z)\right], \tag{11}$$

where $\alpha_0 = [\mu_0\omega\sigma_{xy}^{(0)}]^{1/2} = (ne\mu_0\omega/B)^{1/2}$ is the classical (collisionless) helicon propagation constant, Δ_{xy} is defined in Eq. (10) and $\Delta_{xx} \equiv [\sigma_{xx}/\sigma_{xy}^{(0)}]^2$. All but the leading term in Eq. (11) are directly affected by orbital quantization.

The effect of quantum oscillations and the relative importance of the various contributions can be clearly shown by considering the simple case of high quantum numbers and high damping. Then $f_{\rm osc} \ll 1$ and $\beta z \gg 1$. Substituting Eqs. (9) and (10) into Eq. (11) we obtain (neglecting $2/\beta z$ and higher powers of $f_{\rm osc}$)

$$\varphi_h \simeq \alpha_0 z\left(1 - \frac{1}{2}\frac{(1 + \frac{7}{2}f_{\rm osc})}{(\omega_c\tau_0)^2} + \frac{1}{8}\frac{(1 + \frac{5}{2}f_{\rm osc})^2}{(\omega_c\tau_0)^2}\left(1 - \frac{2}{\beta z}\right)\right)$$

$$= \alpha_0 z\left[1 - \frac{1}{(\omega_c\tau_0)^2}, \frac{9}{8}f_{osc}\right],$$

$$= (\omega\mu_0)^{1/2} z\left[\sigma_{xy}^{(0)}\left(1 - \frac{9}{4}\frac{f_{osc}}{(\omega_c\tau_0)^2}\right)\right]^{1/2}, \tag{12}$$

where the monotonic classical scattering correction term $\frac{3}{8}(\omega_c\tau_0)^{-2}$ has also been omitted. Note that the oscillatory contributions from σ_{xx} and σ_{xy} enter with opposite sign, and the resultant oscillatory behavior of φ_h is dominated by the oscillatory part of σ_{xy}. The phase of the oscillation is thus opposite to that of σ_{xx} (i.e., when σ_{xx} and β display maxima, α will display minima).

The results obtained for high quantum numbers (small $f_{\rm osc}$), in particular the relative importance of the contributions from ϵ' and ϵ'', are expected to remain qualitatively true for lower quantum numbers, which are of direct interest in this paper. Unfortunately the regime of low quantum numbers does not lend itself to similar quantitative analysis in terms of the present model. It is immediately evident from Eqs. (9) and (10) that, as the amplitude of the oscillations increases with increasing magnetic field, the higher powers of $f_{\rm osc}$ must be taken into account. The situation is further complicated by the fact that, when scattering is considered, these terms differ in phase by an amount which in this range depends on the dominant scattering mechanism.[12] Nevertheless, two useful conclusions can immediately be drawn. As long as f_{osc} is of the order of unity (which appears to be typical of quantum oscillations in semiconductors at low quantum numbers, as can be readily inferred from the amplitude of the oscillations in experimentally observed dc behavior of σ_{xx} in InSb),[14] then, by generalizing Eq. (12) to include all orders of f_{osc}, it is still possible to show that the quantum oscillations in the transmitted helicon phase arise predominantly from the oscillatory part of σ_{xy}.[15]

Furthermore, by using the Poisson sum formula to express the oscillatory terms as a harmonic series,[5] it can be shown that the phase shift between σ_{xx} and σ_{xy} arising from collision broadening will not exceed $\frac{1}{4}\pi$. We take this to mean that $(\sigma_{xx})_{max}$ and $(\sigma_{xy})_{min}$ will still occur at approximately the same fields. However, the theoretical formulation of the general behavior of σ_{xx} and σ_{xy} is, at low quantum numbers and intermediate values of f_{osc}, particularly complicated and physically unrevealing, even after drastic simplifying assumptions.

We can, however, substantiate and extend the above conclusions pertinent to this range by an alternate argument, as follows. As the quantum limit is approached, the concepts of periodicity and phase of the quantum oscillations lose their significance, and it is more meaningful to concern oneself with the actual magnetic field values at which magnetoconductivity extrema occur. These are now affected by the electron spin and the magnetic field dependence of the Fermi level. Ideally σ_{xx} maxima correspond to maxima in the scattering rates, which occur when a particular Landau level of quantum number I passes the (magnetic field dependent) Fermi level, i.e., when [16]

$$\zeta_F = (I+\tfrac{1}{2})\hbar\omega_c + \tfrac{1}{2}S\,|g|\,\mu_B B. \qquad (13)$$

Here μ_B is the Bohr magneton, $|g|$ is the magnitude of the g factor for conduction electrons, and S takes the values +1 and −1 for the two spin orientations. Magnetic fields satisfying Eq. (13) are

$$B_I^+ = \frac{2\hbar}{e}\left\{\frac{2}{\pi^2 n}\sum_{k=0}^{I}\left[k^{1/2}+\left(k+\frac{m^*|g|}{2m_0}\right)^{1/2}\right]\right\}^{-2/3}, \qquad (14a)$$

$$B_I^- = \frac{2\hbar}{e}\left\{\frac{2}{\pi^2 n}\sum_{k=1}^{I}\left[k^{1/2}+\left(k-\frac{m^*|g|}{2m_0}\right)^{1/2}\right]\right\}^{-2/3}, \qquad (14b)$$

where m_0 is the free-electron mass. It is easily seen that scattering, which is the very mechanism responsible for the existence of σ_{xx}, *impedes* the current transverse to the electric field which, in the absence of collisions, is described by the ideal Hall conductivity $\sigma_{xy} = ne/B$. It may be said, inferring from the orbit–center migration approach of Kubo *et al.*,[7] that at high magnetic fields the conductivity (dissipative) current exists at the small expense of the Hall (dissipationless) current. We thus expect that *minima* in σ_{xy} should ideally coincide with, and in practice at least lie close to, σ_{xx} maxima. In this respect the present intuitive picture agrees essentially with our conclusions discussed in preceding paragraphs. We thus expect the quantity $\alpha B^{1/2}$, where α is the helicon propagation constant (in which the oscillatory terms are dominated by σ_{xy}), to display minima roughly coincident with absorption maxima.

We remark parenthetically that, as shown in the Appendix, in the related problem of the dc Hall effect we expect the oscillatory contribution of σ_{xy} to be overshadowed by the competing terms in σ_{xx}, so that ideally R_H will display *minima* near σ_{xx} maxima. On this point we differ with GZ. It should be mentioned, however, that the competing contributions of σ_{xx} and σ_{xy} to R_H are expected to be much closer in magnitude than their respective contributions to α, and the discussion of positions of extrema is in the case of the Hall effect far less meaningful, particularly in the presence of significant collision broadening.

EXPERIMENTAL RESULTS

Detailed helicon-phase measurements were carried out as a function of magnetic field on several samples of doped n-type InSb and InAs using a 35-GHz Rayleigh interference bridge.[17] The experiments were carried out at 4.2 °K in the Faraday configuration (propagation parallel to the external magnetic field) in magnetic fields up to 60 kG. The sample parameter range satisfied the helicon limit $\omega_p^2/\omega \gg \omega_c \gg \omega, \tau^{-1}$ as well as the condition $\zeta_F(0) \gtrsim \hbar\omega_c \gg kT$, where $\zeta_F(0)$ is the value of the Fermi energy at B = 0.

The amplitude of the transmitted signal displays the well-known oscillatory behavior associated with quantum effects on helicon damping,[1,2] as shown in Fig. 1. The samples were sufficiently thick (approximately 1–2 mm) so that complications arising from Fabry-Perot–like multiple reflection effects could be ignored, validating the use of Eq. (1) and the neglect of the phase correction term β/α [or, equivalently, the term $2/\beta z$ in Eq. (12)].

In these relatively thick samples a single quantum oscillation spans several helicon-phase cycles, which permits us to resolve the departure of the helicon phase from the ideal behavior. Here we present two superimposed interference curves obtained at two bridge settings 180° out of phase. Thus, the field increment between two consecutive crossover points corresponds to an advance of 180° in the phase of the transmitted helicon signal. A large number of reliable helicon-phase points within a single quantum oscillation can be obtained by recording interference patterns at additional settings of the bridge, e.g., 90° apart (see, e.g., Fig. 1 of Ref. 2). We remark that the crossover points provide a more reliable phase measure than do the interference extrema, which are broader and are, furthermore, subject to shift because of the varying amplitude. This feature is particularly important in dealing with small dispersive anomalies, as in the present case.

Helicon dispersion data are analyzed conventionally by exploiting the fact that the deviation of α from the ideal dependence

$$\alpha_0 = (\omega\mu_0 ne/B)^{1/2} \qquad (15)$$

is very small. In the absence of oscillatory effects, consecutive integers associated with equal increments of advancing phase relative to some arbitrary point, when plotted against corresponding values of $B^{-1/2}$, form a straight line.[17] Assuming phase changes due to interface reflections to be small compared to αz for thick samples, the slope of the resulting plot yields α and n. In Fig. 2 this analysis is applied to phase data obtained for a sample of InSb at 77 °K, where quantum oscillatory effects are absent, yielding a concentration of $n = 1.1\times10^{16}$ cm^{-3}.

A similar plot of phase increments for the data observed on the same sample at 4.2 °K is shown in Fig. 3. The points correspond to equal phase increments of 90°, while the solid line gives α at 77 °K obtained from Fig. 2, which we take as the "ideal" value given by Eq. (15), since the concentration does not change between the two temperatures. We note that the variation of α with $B^{-1/2}$ at 4.2 °K manifests a small but quite discernible and systematic oscillation about the simple classical behavior.

We mention as a point of interest that the above approach to the analysis of the phase dependence on the magnetic field, and particularly its deviation from the ideal behavior, is very similar to that used in the study of the quantum oscillations of the Alfvén wave dispersion in bismuth.[18] Figure 3 is, in fact, strikingly similar to Fig. 3 of Ref. 18. It should of course be emphasized that the mechanisms leading to the oscillatory behavior are in the two cases entirely different. In the present case of a highly doped semiconductor at low temperatures, the carrier concentration is taken as fixed, and the observed oscillations arise entirely from the influence of the magnetic field on scattering. The mechanism giving rise to the quantum oscillations of the Alfvén velocity in Bi is, on the other hand, associated with the explicit fluctuation of the carrier density which takes place in this compensated system $(n_h = n_e)$ as a consequence of the magnetic field dependence of ζ_F. In

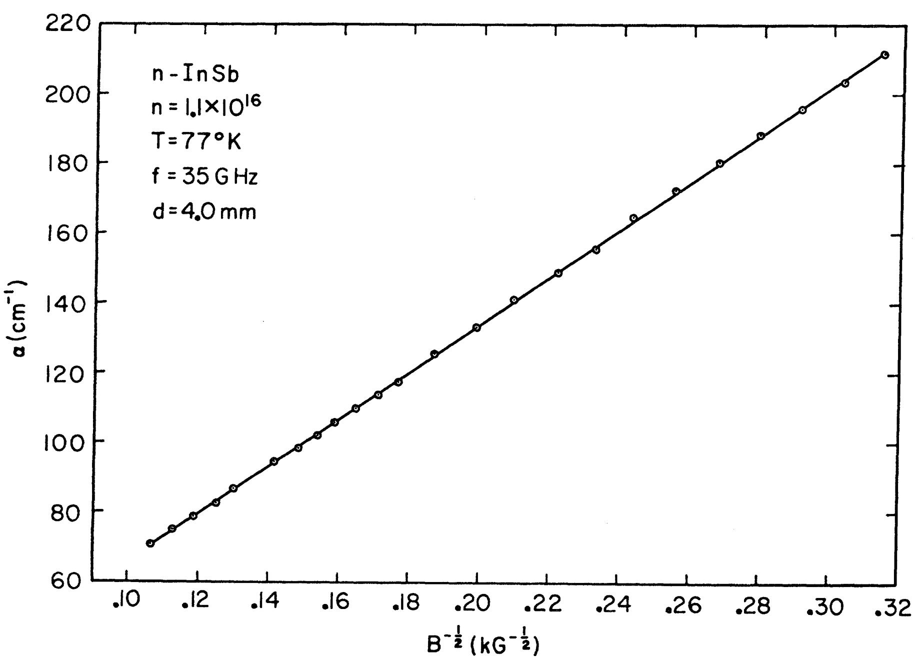

FIG. 2. An integer plot of α versus $B^{-1/2}$ for helicon transmission at 77 °K observed with the same sample as in Fig. 1. The data show a purely classical behavior in accord with Eq. (15) and provide a measure of the concentration.

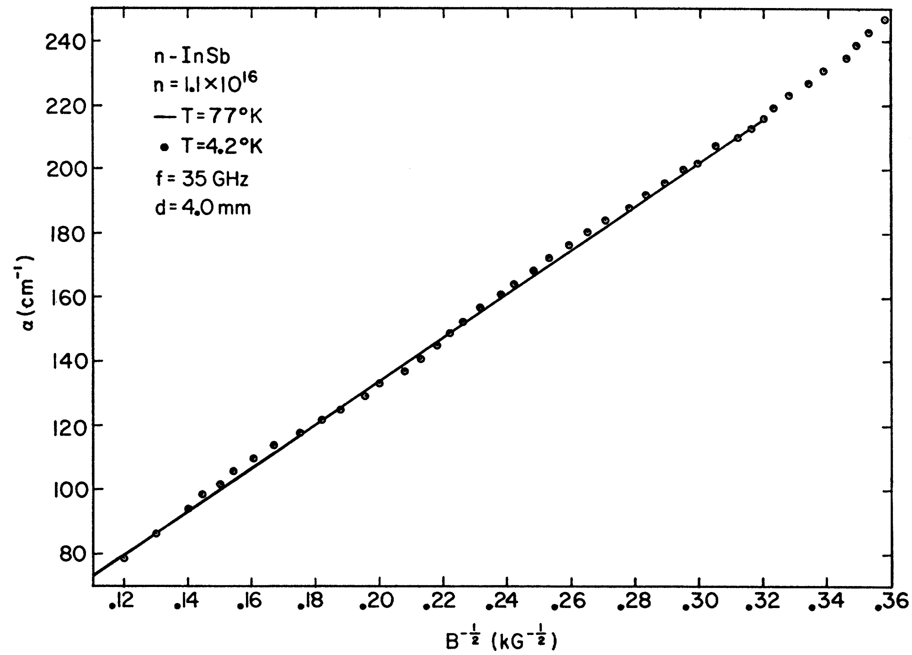

FIG. 3. Helicon-phase constant versus $B^{-1/2}$ for helicon transmission data on n-type InSb. The points represent transmission at 4.2 °K obtained from the data of Fig. 1, showing phase behavior in quantizing fields. Helicon phase transmitted in the quantum range displays unambiguous oscillations about the classical behavior of 77 °K, represented by the solid line taken from Fig. 2.

principle, Alfvén wave velocity would also be influenced by *scattering* oscillations of the type investigated in the present paper, but for very large values of $\omega_c\tau$ this oscillatory contribution to the dispersion is presumably masked by the aforementioned effect involving the density fluctuation.

To display the oscillatory nature of φ_h, we plot in Fig. 4 the normalized quantity $\varphi_h B^{1/2}/z \simeq \alpha B^{1/2}$, obtained from the 4.2 °K data. We observe a clear oscillatory behavior, considerably in excess of experimental error. The liquid-nitrogen data, which did not depart from the classical behavior within experimental error, are shown by the dashed line for comparison. The experimentally observed positions of maximum helicon damping for the same sample are indicated for the last two singularities. The field B_1 is in excellent agreement with Eq. (14), while the last experimentally observed transmission dip marked B_0^* occurs consistently at fields lower than those predicted by Eq. (14) (see Fig. 1 and Refs. 1 and 2). It is clear that $\alpha B^{1/2}$ displays *minima* near the absorption maxima, which seems to indicate that oscillations in α are indeed dominated by the oscillatory part of σ_{xy}, in accord with the preceding discussion. It would be premature, however, to draw any further conclusions of precise quantitative nature regarding the relative phases of the two oscillatory phenomena, both on account of the size of the experimental error and of the present state of the theory.

Similar data, obtained on doped n-type InAs and shown in Fig. 5, display essentially the same features. The fields B_1 and B_2 indicate positions of maximum absorption, which in turn agree well with the theoretical values obtained with Eq. (14). The normalized phase $\alpha B^{1/2}$ shows somewhat smaller (of the order of 1%) but quite unambiguous oscillations. The minima again occur in the vicinity of the σ_{xx} singularities.

The order of magnitude of the effect at fields $B < B_0^*$ in both materials is somewhat larger than that typically seen in the quantum oscillations of the Hall coefficient at these quantum numbers,[19] probably because in R_H the competing contributions associated with $(\Delta_{xy})_{osc}$ and $(\sigma_{xx})_{osc}$ are closer in magnitude (see Appendix).

CONCLUDING REMARKS

We have shown that, in the local helicon limit, the major effect of orbital quantization on helicon dispersion enters through the dc value of the collision-dependent correction $\Delta_{xy} - \frac{1}{4}[\sigma_{xx}/\sigma_{xy}^{(0)}]^2$ in Eq. (11). The results of GZ[5] indicate that the competing quantities Δ_{xy} and $(\sigma_{xx}/\sigma_{xy}^{(0)})^2$, which in the classical constant-τ model are both equal to $(\omega_c\tau)^{-2}$, are also comparable in magnitude in the degenerate quantum model. We conclude on this basis that oscillations in the transmitted helicon phase will be dominated by $(\Delta_{xy})_{osc}$, defined in Eq. (10). It was shown that in the range of high quantum numbers the oscillations of σ_{xx} and Δ_{xy} have opposite phase, and thus β_{max} will coincide with $(B^{1/2}\alpha)_{min}$.

In the region of lowest quantum numbers the concept of period and phase of the oscillations lose their usefulness, and it is more meaningful to discuss the positions of extrema (e.g. the σ_{xx} maxima) associated with individual Landau levels. We find the approach of GZ unsuitable for locating the extrema in this region. However, by referring directly to the physical mechanism of the oscillations in σ_{xx} and σ_{xy}, i.e. to the sharp maxima in scattering rates at the quantum resonances, we again conclude that the absorption maxima and the small minima in $\alpha B^{1/2}$ should occur, at least approximately, at the same fields.

This coincidence appears to be confirmed by our experimental results observed in several samples of n-type InSb and InAs. The oscillations in $\alpha B^{1/2}$ are of the order of 1–2% and, while small, are quite unambiguous. At this stage, however, in the absence of suitable theory for relative amplitudes of the oscillatory terms in the low quantum-number region, we make no attempt at a quantitative interpretation of the magnitude of the effect.

It is interesting to compare the oscillatory nature of helicon dispersion with that of the dc Hall coefficient R_H. The competing oscillatory terms in the latter are closer in magnitude (see Appendix), making the oscillatory character of R_H weaker and the individual contributions of σ_{xx} and Δ_{xy} harder to disentangle. In the light of the model adopted here we note that the oscillatory character of R_H is, unlike α, dominated by the σ_{xx} contribution. It would appear, then, that the helicon dispersion affords a particularly useful tool for the study of the effects of scattering processes on the dissipationless Hall conductivity σ_{xy}.

Our discussion of the oscillatory nature of helicon dispersion does not include two effects which may prove significant under certain circumstances. First, we have neglected nonlocal effects. These can be shown to be quite small in the parameter range corresponding to our experimental work characterized by the inequality $kR \ll kl \ll 1$, where $l = v_F \tau$ is the mean free path, v_F is the Fermi velocity, and $R = v_F/\omega_c$ is the cyclotron radius. Under these conditions and for wave propagation *along* the magnetic field, the contribution of nonlocal effects to σ_{xx} and σ_{xy} is of the order of $(kR)^2$.[20] Thus, the effect of spatial dispersion on σ_{xx} can immediately be ignored. The importance of the nonlocal correction to σ_{xy} is, in the context of this paper, to be compared to the oscillatory part Δ_{xy}. The nonlocal contribution to the Hall conductivity for our parameter range has been shown by Sheard[21] to be $\frac{1}{5}\sigma_{xy}^{(0)} k^2R^2$. The relative magnitude of nonlocal effects relative to the Shubnikov–de Haas oscillations of σ_{xy} are then estimated by

$$\tfrac{1}{5}\sigma_{xy}^{(0)} k^2R^2/[\sigma_{xy}^{(0)} \Delta_{xy}] \simeq \tfrac{1}{5}(kv_F\,\tau/\omega_c\tau)^2(\omega_c\tau)^2 \simeq \tfrac{1}{5}kl,$$

which for our purposes is still a very small number. Thus, effects of spatial dispersion can be safely neglected throughout this paper. It is obvious, however, that when $kl > 1$ (which is possible even when the high-field conductivity is essentially "local," i.e., $kR \ll 1$), the problem of quantum oscillations in helicon phase must be reexamined and should be of considerable interest.[22]

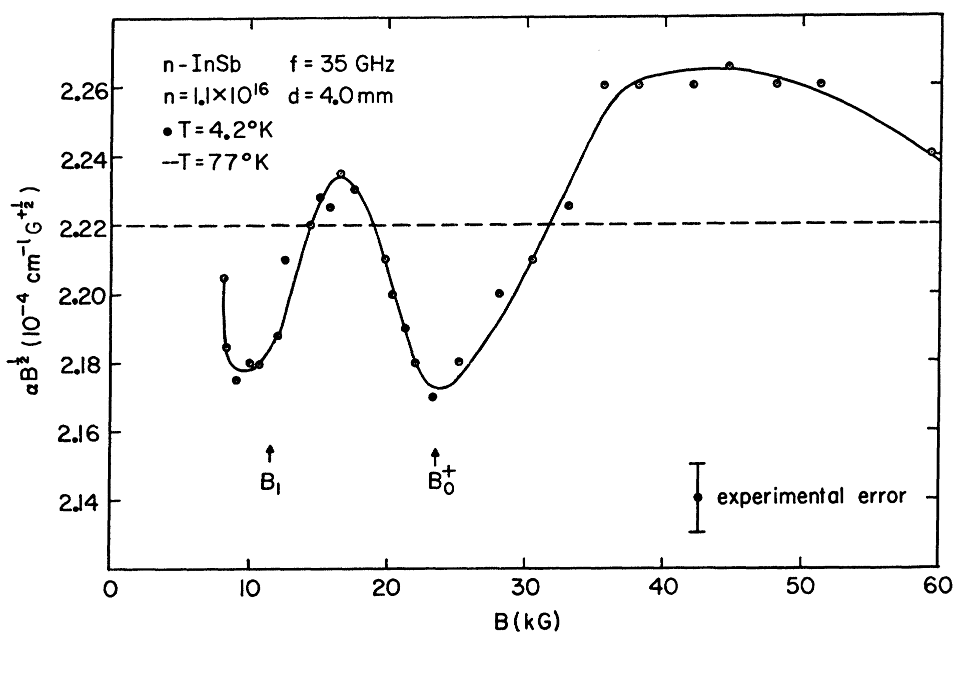

FIG. 4. Normalized helicon phase versus B for quantizing fields, showing quantum oscillations. The points with the solid curve drawn through them correspond to the 4.2 °K data of Figs. 1 and 3. The dashed line represents 77 °K data. Fields corresponding to experimentally observed helicon damping maxima for the same sample, as well as the experimental error in the transmitted phase, are indicated.

Second, we have assumed throughout this discussion that the quantum oscillations in transport parameters arise primarily from the oscillatory character of the scattering process, and we have assumed that the electron concentration remains constant as the magnetic field is varied. Recently, however, the possibility of a perceptible fluctuation in n itself as a function of B has been raised in connection with anomalous deviations observed in the Hall coefficient at fields just above the last quantum oscillation, at least in fairly pure samples.[14] The mechanism for this possible fluctuation is not clearly understood at present. We suggest that simultaneous study of the oscillatory behavior of R_H and α, which depend identically on n but differently on σ_{xx} and Δ_{xy}, may serve in resolving this intriguing and rather fundamental question.

We remark finally that the relationship between the quantum oscillations in helicon dispersion and damping should, in principle, be contained in the Kramers-Kronig relations connecting ϵ' and ϵ''. Unfortunately, in order to exploit this otherwise powerful technique to obtain, e.g., the complete form of ϵ' at a particular value of ω, presupposes the explicit knowledge of ϵ'' at *all* values of ω. This, in the absence of a convenient analytic formulation for the frequency-dependent conductivity of a quantum degenerate plasma and in the absence of measurements over a wide frequency spectrum, does not appear practical at present. It is nevertheless a problem of considerable interest and hopefully its theoretical as well as experimental aspects will eventually be resolved. At the moment we must emphasize that the approximate helicon-limit expressions for ϵ' and ϵ'', Eqs. (5) and (7), will not as they stand satisfy the Kramers-Kronig relations, since both are obtained for a restricted frequency range.

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APPENDIX

It is instructive to discuss in this context the oscillatory behavior of the dc Hall coefficient R_H which, in zeroth order, constitutes a direct analog to the helicon propagation constant α. The relationship between the two quantities is borne out clearly by examining R_H under conditions corresponding to Eq. (12), which gives α for large quantum numbers:

$$R_H B = \rho_{xy} = \sigma_{xy}/(\sigma_{xx}^2+\sigma_{xy}^2) \simeq [\sigma_{xy}(1+\Delta_{xx})]^{-1}$$

$$= \left[\sigma_{xy}^{(0)}\left(1-\frac{1+\frac{7}{2}f_{osc}}{(\omega_c\tau_o)^2}\right)\left(1+\frac{(1+\frac{5}{2}f_{osc})^2}{(\omega_c\tau_o)^2}\right)\right]^{-1}$$

$$= \left[\sigma_{xy}^{(0)}\left(1+\frac{3}{2}\frac{f_{osc}}{(\omega_c\tau_c)^2}\right)\right]^{-1}. \qquad \text{(A1)}$$

Thus, in the high quantum-number region the oscillations of the Hall coefficient are dominated by the term σ_{xx}, which opposes the oscillatory part of σ_{xy} and determines the phase of the oscillation.

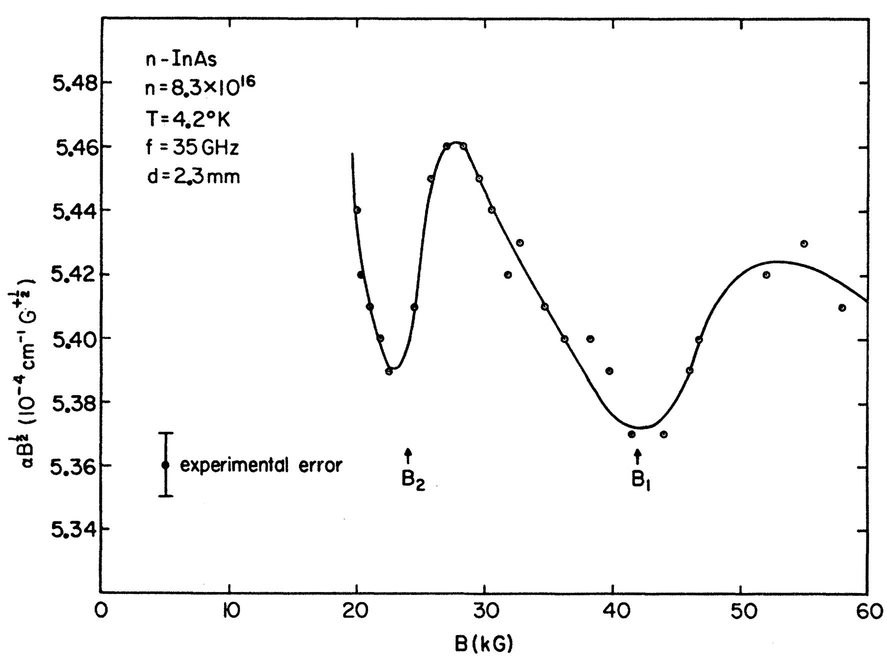

FIG. 5. Normalized helicon-phase data versus B obtained in quantizing fields for n-type InAs. Experimental fields corresponding to maximum helicon damping, which agree well with theoretical values given by Eq. (14) as well as the experimental error involved, are indicated.

We remark in passing that our conclusion regarding the phase of the Hall coefficient is then opposite to that reached by GZ, who appear to neglect the term Δ_{xx} in their development. It is interesting to note that the classical collision term $(\omega_c \tau_0)^{-2}$ cancels identically in the case of R_H, a result which remains true even for low quantum numbers within the framework of the GZ model. Since, except for the leading collisionless terms, helicon dispersion and the Hall effect are *different* functions of the conductivity, simultaneous measurement of both may serve in clarifying the role of scattering on the quantity σ_{xy} and the entire matter of relative phases of oscillatory phenomena.

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